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**RAN-2103000206020035****T.Y.B.Sc. (Sem. VI) Examination April - 2023****Mathematics : MTH-605 - Discrete Mathematics****[ Total Marks: 50****સૂચના : / Instructions**

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નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.  
Fill up strictly the details of signs on your answer book

Name of the Examination:

T.Y.B.Sc. (Sem. VI)

Name of the Subject :

Mathematics : MTH-605 - Discrete Mathematics

Subject Code No.: 2103000206020035

Seat No.:

Student's Signature

- (2) All Questions are compulsory.  
(3) Figures to the right indicate marks of the corresponding question.  
(4) Follow usual notations.

**Q:1 Answer the following. (Any five)****10**

1. Give Truth table representation of the function  $f(x, y, z) = x\bar{y} + \bar{y}z$
2. Let  $\rho(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$  and  $B = \{\{b\}, \{c\}, \{b, c\}\}$ . Find upper bound and lower bound for B.
3. Let  $P = \{2, 4, 6, 8\}$ . If  $\langle P, D \rangle$  be a Partially ordered set then find cover of elements 2 and 4 in it.
4. Let  $L_1$  and  $L_2$  are chain of divisors of 4 and 9 respectively. Draw the Hasse diagram of  $L_1 \times L_2$ .
5. In any Boolean algebra B, prove that  $(a + b')(a' + b) = ab + b'a' \forall a, b, c \in B$
6. Let  $\leq$  and  $D$  two relations on a set  $A = \{1, 3, 6, 9, 12\}$ . Find  $\leq \cap D$ .
7. Obtain the simplified Boolean expression which is equivalent to the expression  $\max_8 * \max_{15} * \max_{12}$ .
8. Draw Hasse Diagram of  $\langle S_{24}, D \rangle$  and write any four sub lattices of it.

- Q:2 Answer the following. (Any Two) 10**
1. State and prove Isotonicity property of a lattice.
  2. If  $x = \{1,2,3,4,5,6,7\}$  and the relation  $R = \{\langle x, y \rangle / x - y \text{ is divisible by } 4\}$  define on X then Show that R is an Equivalence relation.  
Also Draw the sketch its graph.
  3. Let  $I_+$  be the set of all positive integers and let D denotes the relation of 'division' in  $I_+$ , such that for any  $a, b \in I_+$ ,  $a D_b$  if and only if 'a divides b'. Then show that  $\langle I_+, D \rangle$  is a lattice with  $a \oplus b = \text{LCM} \{a, b\}$  and  $a * b = \text{GCD} \{a, b\}$ .
- Q:3 Answer the following. (Any Two) 10**
1. Is  $\langle S_{75}, D \rangle$  Complemented Distributive Lattice? Justify your answer.
  2. Define Lattice Isomorphism. Let  $\langle L, *, \oplus \rangle$  and  $\langle S, \wedge, \vee \rangle$  be two lattices, with partial ordering relation  $\leq$  and  $\leq'$  respectively. If  $g : L \rightarrow S$  is an isomorphism then show that  $a \leq b \Leftrightarrow g(a) \leq' g(b)$ .
  3. Show that in a complemented distributive lattice  
 $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$
- Q:4 Answer the following. (Any Two) 10**
1. Simplify the following Boolean Expressions:  
(i)  $a' * [(b' \oplus c)' \oplus (b * c)] \oplus [(a \oplus b')' * c]$   
(ii)  $[a * (b' \oplus c)]' * [b' \oplus (a * c)]'$
  2. Let  $\langle B, *, \oplus, ', 0, 1 \rangle$  and  $\langle P, \cap, \cup, -, \alpha, \beta \rangle$  be two Boolean Algebras. If  $f: B \rightarrow P$  is a mapping preserves the operations  $*$  and  $'$  then show that  $f: B \rightarrow P$  is a Boolean Homomorphism.
  3. Obtain the product of sums canonical form of  $x_1 * x_2$  in three variables  $x_1, x_2, x_3$ .
- Q:5 Answer the following. (Any Two) 10**
1. Use the Karnaugh map representation to find a minimal sum of products expression of the function  $f(a,b,c) = \sum(5,7,10,13,15)$
  2. Use the Quine-Mc-Cluskey method to find a minimal sum of products expression of the function  $f(a,b,c,d) = \sum(0,5,7,8,12,14)$
  3. Give cubic and Karnaugh map representation of the Boolean function  $f(a,b,c) = \sum(0,1,4,6)$ .